WEEKLY TEST RANKER'S BATCH TEST - 09 RAJPUR SOLUTION Date 24-11-2019

[PHYSICS]

1. **(b)** Acceleration of simple harmonic motion is

$$a_{max} = -\omega^2 A$$

or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{\omega_1^2}{\omega_2^2}$ (as A remains the same)

or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1:10^2$

2. **(d)** Velocity is the time derivative of displacement.

Writing the given equation of a point performing SHM

$$x = a\sin\left(\omega t + \frac{\pi}{6}\right) \quad \ldots(i)$$

Differentiating Eq. (i), w.r.t. time, we obtain

$$v = \frac{dx}{dt} = a\,\omega\cos\left(\omega t + \frac{\pi}{6}\right)$$

It is given that $v = \frac{a\omega}{2}$, so that

$$\frac{a\omega}{2} = a\,\omega\cos\left(\omega t + \frac{\pi}{6}\right)$$

or $\frac{1}{2} = \cos\left(\omega t + \frac{\pi}{6}\right)$

or $\cos\frac{\pi}{3} = \cos\left(\omega t + \frac{\pi}{6}\right)$

or $\omega t + \frac{\pi}{6} = \frac{\pi}{3} \Rightarrow \omega t + \frac{\pi}{6}$

or $t = \frac{\pi}{6\omega} = \frac{\pi \times T}{6 \times 2\pi} = \frac{T}{12}$

Thus, at $T/12$ velocity of the point will be equal to half of its maximum velocity.

3. **(d)** $v = \frac{dy}{dt} = A\omega\cos\omega t = A\omega\sqrt{1-\sin^2\omega t}$

$$= \omega\sqrt{A^2 - y^2}$$

Here, $y = \frac{a}{2}$

$\therefore\ v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3a^2}{4}} = \frac{2\pi}{T}\frac{a\sqrt{3}}{2} = \frac{\pi a\sqrt{3}}{T}$

4. **(b)** Acceleration $\propto$ – (displacement).

$$A \propto -y$$

$$A = -\omega^2 y$$

$$A = -\frac{k}{m}y$$

$$A = -ky$$

Here, $y = x + a$

$\therefore$ acceleration $= -k(x+a)$

5. **(b)** Use the law of conservation of energy. Let x be the extension in the spring.

Applying conservation of energy

$$mgx - \frac{1}{2}kx^2 = 0 - 0 \Rightarrow x = \frac{2mg}{k}$$

6. **(c)** For a particle executing SHM

acceleration $a \propto -\omega^2$ displacement (x) ...(i)

Given $x = a\sin^2 \omega t$ ...(ii)

Differentiating the above equation, we get

$$\frac{dx}{dt} = 2a\omega(\sin\omega t)(\cos\omega t)$$

Again differentiating, we get

$$\frac{d^2x}{dt^2} = a = 2a\omega^2\left[\cos^2\omega t - \sin^2\omega t\right]$$

$$= 2a\omega^2\cos 2\omega t$$

The given equation does not satisfy the condition for SHM [Eq. (i)]. Therefore, motion is not simple harmonic.

7. **(d)** Time period of spring pendulum, $T = 2\pi\sqrt{\frac{M}{k}}$.

If now mass in doubled $T' = 2\pi\sqrt{\frac{2M}{k}} = \sqrt{2}T$

8. **(d)** The given velocity-position graph depicts that the motion of the particle is SHM.

In SHM, at $t = 0$, $v = 0$ and $x = x_{max}$.

So, option (d) is correct.

9. **(b)** For a simple harmonic motion $\frac{d^2y}{dt^2} \propto -y$

Hence, equation $y = \sin\omega t - \cos\omega t$ and $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$ satisfy this condition and equation $y = 1 + \omega t + \omega^2t^2$ is not periodic and $y = \sin^3\omega t$ is periodic but not SHM.

Option (c) is correct.

10. **(b)** Equation of SHM is given by

$$x = A\sin(\omega t + \delta)$$

$(\omega t = \delta)$ is called phase.

when $x = \frac{A}{2}$, then

$$\sin(\omega t + \delta) = \frac{1}{2} \Rightarrow \omega t + \delta = \frac{\pi}{6}$$

or $\phi_1 = \frac{\pi}{6}$

For second particle, $\phi_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\therefore \quad \phi = \phi_2 - \phi_1$

$$= \frac{4\pi}{6} = \frac{2\pi}{3}$$

11. **(a)** Given, damping force $\propto$ velocity

$$F = kv \Rightarrow k = \frac{F}{v}$$

$$\text{Unit of } k = \frac{\text{unit of } F}{\text{unit of } v} = \frac{\text{kg-ms}^{-2}}{\text{ms}^{-1}} = \text{kgs}^{-1}$$

12. **(b)** We now that $v_{max} = a\omega$ and $v = n\lambda$

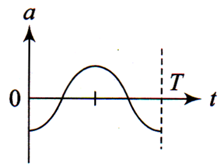
$$\therefore \quad \frac{v_{max}}{v} = \frac{a\omega}{n\lambda} = \frac{a(2\pi n)}{n\pi} = \frac{2\pi a}{\lambda}$$

$$= \frac{2\pi a}{2\pi/k} = ka = \frac{\pi}{2} \times 3 = \frac{3\pi}{2}$$

13. **(c)** Displacement, $x = A\cos(\omega t)$ (given)

Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t)$

Acceleration, $a = \frac{dv}{dt} = -A\omega^2\cos(\omega t)$



Hence graph (c) correctly depicts the variation of a with t.

14. **(c)** The two displacement equations are $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t) = b\sin\left(\omega t + \frac{\pi}{2}\right)$

$$y_{eq} = y_1 + y_2$$

$$= a\sin\omega t + b\cos\omega t$$

$$= a\sin\omega t + b\sin\left(\omega t + \frac{\pi}{2}\right)$$

Since the frequencies for both SHMs are same, resultant motion will be SHM.

Now $A_{eq} = \sqrt{a^2 + b^2 + 2ab\cos\frac{\pi}{2}}$

$$\Rightarrow \quad A_{eq} = \sqrt{a^2 + b^2}$$

15. **(b)** As we know, for particle undergoing SHM,

$$V = \omega\sqrt{A^2 - X^2}$$

$$V_1^2 = \omega^2\left(A^2 - x_1^2\right) \text{ and } V_2^2 = \omega^2\left(A^2 - x_2^2\right)$$

On subtracting the relations

$$V_1^2 - V_2^2 = \omega^2\left(x_2^2 - x_1^2\right)$$

$$\omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$$

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$$

16. **(a)** Maximum velocity $V_{\max} = A\omega = \beta$ **(i)**

maximum acceleration $\alpha_{\max} = A\omega^2 = \alpha$ **(ii)**

Equation (ii) divided by (i) $\omega = \frac{\omega}{\beta} \Rightarrow \frac{2\pi}{T} = \frac{\omega}{\beta}$

$$T = \frac{2\pi\beta}{\alpha}$$

17. **(a)** If initial length $l_1 = 100$ then $l_2 = 121$

By using $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

Hence, $\frac{T_1}{T_2} = \sqrt{\frac{100}{121}} \Rightarrow T_2 = 1.1T_1$

$$\% \text{ increase} = \frac{T_2 - T_1}{T_1} \times 100 = 10\%$$

Alternative: Time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

Since, $\frac{\Delta l}{l} = 21\%$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2} \times 21\% \approx 10\%$$

18. **(d)** As springs are connected in series, effective force constant

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

19. **(b)** Time taken by particle to move from $x = 0$ (mean position) to $x = 4$ (extreme position) $= \frac{T}{4} = \frac{1.2}{4} = 0.3$ sec

Let t be the time taken by the particle to move from $x = 0$ to $x = 2$ cm

$$y = a\sin\omega t \Rightarrow 2 = 4\sin\frac{2\pi}{T}t \Rightarrow \frac{1}{2} = \sin\frac{2\pi}{1.2}t$$

$$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{1.2}t \Rightarrow t = 0.1 \text{ sec}$$

Hence time to move from $x = 2$ to $x = 4$ will be equal to $0.3 - 0.1 = 0.2$ sec

Hence total time to move from $x = 2$ to $x = 4$ and back again $= 2 \times 0.2 = 0.4$ sec

20. **(c)** $n = \frac{1}{2\pi}\sqrt{\frac{K_{\text{effective}}}{m}}$

Springs are connected in parallel

$$K_{\text{eff}} = K_1 + K_2 = K + 2K = 3K$$

$$\Rightarrow \quad n = \frac{1}{2\pi}\sqrt{\frac{(K + 2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$$

21. **(a)** As springs are connected in series, effective force constant

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \Rightarrow k_{\text{eff}} = \frac{k}{2}$$

Hence, frequency of oscillation is

$$n = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2M}}$$

22. **(a)** Let $y = \sin\omega t - \cos\omega t$

$$\frac{dy}{dt} = \omega\cos\omega t + \omega\sin\omega t$$

$$\frac{d^2y}{dt^2} = -\omega^2\sin\omega t + \omega^2\cos\omega t$$

or $\quad a = -\omega^2(\sin\omega t - \cos\omega t)$

$$a = -\omega^2 y$$

$\Rightarrow \quad a \propto -y$

This satisfies the condition of SHM. Other equations do not. Hence, $\sin\omega t - \cos\omega t$. represents SHM

23. **(d)** $T \propto \sqrt{m} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{5}{3} = \sqrt{\frac{M+m}{M}}$

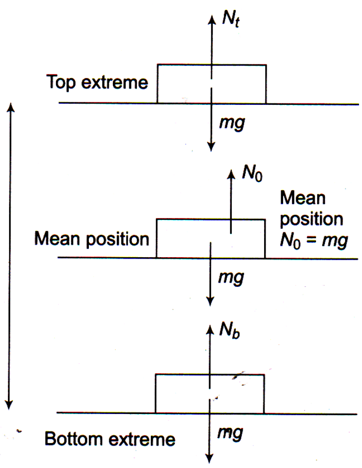
$$\Rightarrow \frac{25}{9} = \frac{M+m}{M} \Rightarrow \frac{m}{M} = \frac{16}{9}$$

24. **(a)** In this case frequency of oscillation is given by $n = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2+a^2}}{l}}$, where a is the acceleration of car. If a increases then n also increases.

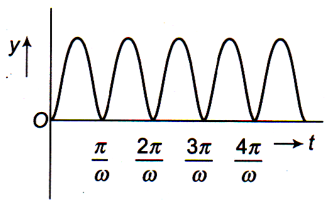
25. **(c)** The net effect of these two forces must be towards mean position.

At the mean position, there is no net force and hence normal reaction equals mg. Above mean position, normal reaction is less than mg and below mean position, normal reaction is greater than mg.

At the top extreme:



26. **(d)** Here, $y = \sin^2 \omega t$



$$\frac{dy}{dt} = 2\omega \sin \omega t \cos \omega t = \omega \sin 2\omega t$$

$$\frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$$

For SHM, $\frac{d^2y}{dt^2} \propto -y$

Hence, function is not SHM, but periodic.

From the y–t graph, time period is $t = \frac{\pi}{\omega}$

27. **(d)** Potential energy $U = k|x|^3$

Hence force, $F = -\frac{dU}{dx} = -3k|x|^2$ ...(i)

Also, for SHM, $x = a \sin \omega t$

and $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\Rightarrow$ Acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2 x$

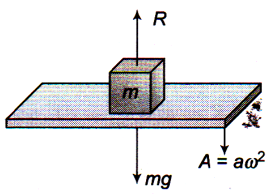
$\Rightarrow$ $F = ma = m\frac{d^2x}{dt^2} = -m\omega^2 x$ ...(ii)

From Eqs. (i) and (ii), we get $\omega = \sqrt{\frac{3kx}{m}}$

$\Rightarrow$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}}$

$\Rightarrow$ $T \propto \frac{1}{\sqrt{a}}$

28. **(a)** By drawing free body diagram of object during the downward motion at extreme position, for equilibrium of mass



$mg - R = mA$ (A = Acceleration)

For critical condition $R = 0$

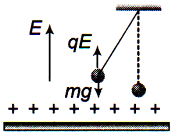
so $mg = mA \Rightarrow mg = ma\omega^2$

$$\Rightarrow \omega = \sqrt{\frac{g}{a}} = \sqrt{\frac{9.8}{3.92\times10^{-3}}} = 50$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1256 \text{ sec}$$

29. **(a)** In this case time period of pendulum becomes

$$T' = 2\pi\sqrt{\frac{l}{\left(g + \frac{qE}{m}\right)}}$$



$\Rightarrow \quad T' < T$

30. **(a)** $T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow m \propto T^2 \Rightarrow \frac{m_2}{m_1} = \frac{T_2^2}{T_1^2}$

$$\Rightarrow \frac{M+m}{M} = \left(\frac{\frac{5}{4}T}{T}\right)^2 \Rightarrow \frac{m}{M} = \frac{9}{16}$$

30. **(c)** $\frac{mv^2}{r} > mg$

At highest point $mg = \frac{mv^2}{r} - N$

31. $K = \frac{1}{2} m\omega^2 (A^2 - y^2), \quad U = \frac{1}{2} m\omega^2 y^2$

$K = U$ or $\frac{1}{2} m\omega^2 (A^2 - y^2) = \frac{1}{2} m\omega^2 y^2$

i.e., $2y^2 = A^2$ or $y = \frac{A}{\sqrt{2}}$.

32. B
33. D

34. According to theory section,

$$f = \frac{1}{2\pi}\sqrt{\frac{BA^2}{MV_0}}$$

$\therefore \quad T = 2\pi\sqrt{\frac{MV_0}{BA^2}} = 2\pi\sqrt{\frac{M(hA)}{BA^2}} = 2\pi\sqrt{\frac{Mh}{BA}}$.

As $B = P$

Hence, $\quad T = 2\pi\sqrt{\frac{Mh}{PA}}$

35. B
36. B
37. D
38. B
39. C
40.

Time taken by the pendulum to move from A to O and from O to $A = \frac{T}{2}$.

Time period of oscillation $\propto \sqrt{L}$.

$\therefore \quad \frac{T_1}{T} = \sqrt{\frac{L/4}{L}} = \frac{1}{2}$ or $T_1 = \frac{T}{2}$

Time taken to complete half the oscillation $= \frac{T}{4}$.

Total time period of oscillation

$$= \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}.$$

41. B
42. A
43. A
44. A
45. For simple harmonic motion, $v = \omega\sqrt{a^2 - x^2}$

When $x = \frac{a}{2}, v = \omega\sqrt{a^2 - \frac{a^2}{4}} = \omega\sqrt{\frac{3}{4}a^2}$

As $\quad \omega = \frac{2\pi}{T},$

$\therefore \quad v = \frac{2\pi}{T}.\frac{\sqrt{3}}{2}a$

or $\quad v = \frac{\pi\sqrt{3}a}{T}$